

# Theoretical evaluation of $k_p$ and $\lambda$ in g.p.c.: A criterion to define ideal reference systems

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From a thermodynamic point of view, theoretical expressions for the distribution coefficient of solute-gel interactions in g.p.c.,  $k_p$ , and the preferential sorption coefficient,  $\lambda$ , have been developed. In the light of these expressions, a criterion to define an ideal reference system in which solutes are separated solely by steric exclusion,  $k_p = 1$ , has been reached. This criterion implies that  $g_{12}^\circ + g_{13}^\circ = 1$  and  $\lambda = 0$  ( $g_{ij}^\circ$  being the Flory-Huggins interaction parameters). From literature data of  $k_p$ ,  $\lambda$  values have been calculated. These values vary with molecular weight as in the case of solvent(1)/solvent(2)/polymer(3) systems.

(**Keywords:** distribution coefficient; solute-gel interactions; sorption coefficient; gel permeation chromatography)

## INTRODUCTION

In g.p.c. on organic packings, the universal calibration concept<sup>1</sup> is well established for different polymers and solvents provided that the solvents are good for the polymers. However, when the polymers are eluted in poor or 'theta' solvents, solute-gel interactions become important and secondary mechanisms appear (adsorption and partition<sup>2-4</sup>) with the retention volume,  $V_R$ , being displaced to high  $V_R$  values with respect to universal calibration. In other cases, because of incompatibility between solute polymer and gel,  $V_R$  values are shifted toward smaller retention volumes<sup>5</sup>.

The adsorption of the solute polymer onto the gel or the opposite case of incompatibility between solute polymer and gel can be related to preferential or selective sorption. In this way, a theoretical expression for the distribution coefficient of solute-gel interactions,  $k_p$ , has been recently derived<sup>5</sup>.

In this paper we present a more extended theoretical procedure to evaluate  $k_p$ , the expressions reached are equivalent to those obtained previously. Furthermore, the thermodynamic treatment has been also applied in order to derive an equation for the preferential sorption coefficient,  $\lambda$ .

The aim of this work is to arrive at a criterion defining a reference system in which solutes are separated solely by steric exclusion,  $k_p = 1$ . This criterion implies that for a solvent(1)/solute polymer(2)/gel(3) system  $g_{13}^\circ + g_{12}^\circ = 1$  ( $g_{ij}^\circ$  being the Flory-Huggins interaction parameters at infinite dilution) and  $\lambda = 0$ .

On the other hand, from literature  $k_p$  values<sup>2-4,6</sup> we obtain  $\lambda$  values. When  $k_p > 1$  (compatibility between the solute polymer and gel) the  $\lambda$  values are negative. These values vary with molecular weight as in the case of solvent(1)/solvent(2)/polymer(3) systems.

## THERMODYNAMIC CALCULATIONS

A polymer chain fractionation by g.p.c. on swelling sorbents can be regarded as a polymeric ternary system, constituted by solvent (1)/polymeric solute (2)/gel matrix (3) (swollen gel). Under usual chromatographic conditions and from a thermodynamic point of view this system can be considered as being formed by two phases coexisting in equilibrium:

(1) A 'binary phase', constituted by the polymer solution outside the pore (mobile phase) or inside the pore but away from the pore walls.

(2) A 'ternary phase' (stationary phase), constituted by the mixture of polymer solution and the gel matrix. Only a little fraction of the crosslinked polymer (gel matrix) is solvated by the solvent taking part in the separation process.

Throughout the text, the following nomenclature will be used to define the composition in both phases:

$\phi_{i0}$  ( $i = 1, 2$ ): the volume fraction of the component  $i$  in the binary phase ( $\phi_{10} + \phi_{20} = 1$ ).

$\phi_i$  ( $i = 1, 2, 3$ ): the volume fraction of the component  $i$  in the ternary phase ( $\phi_1 + \phi_2 + \phi_3 = 1$ ).

$u_i$  ( $i = 1, 2$ ): the volume fraction of the polymer solution (components 1 and 2) in the ternary phase,  $u_i = \phi_i / (1 - \phi_3)$ .

The Flory-Huggins (F-H) expression for the Gibbs free energy of mixing,  $\Delta G_M$ , for a system formed by two polymers and one solvent is<sup>7</sup>:

$$\Delta G_M / RT = \sum_{i=1}^3 n_i \ln \phi_i + (g_{12} \phi_1 \phi_2 + g_{13} \phi_1 \phi_3 + g_{23} \phi_2 \phi_3) \sum_{i=1}^3 m_i n_i \quad (1)$$

and the corresponding expression,  $\Delta G_M^b$ , for a system formed by a polymer and a solvent is

$$\Delta G_M^b / RT = n_1 \ln \phi_{10} + n_2 \ln \phi_{20} + n_1 \phi_{20} g_{12} (\phi_{10}) \quad (2)$$

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where  $n_i$  is the number of moles of component  $i$  in the mixture,  $m_i$  the ratio between molar volumes of component  $i$  and component 1 (reference component) and  $g_{ij}$  the F-H interaction parameter which is related to the classical parameter,  $\chi_{ij}$ , by

$$\chi_{ij}(\phi_j) = g_{ij}(\phi_j) - (1 - \phi_j)(dg_{ij}/d\phi_j) \quad (3)$$

at constant  $P$  and  $T$ .

In order to describe the equilibrium between both phases we can use the thermodynamics of polymer solutions writing<sup>8</sup>

$$(\partial G_u/\partial u_1)_{u_s} = (\partial \Delta G_M/\partial \phi_{10}) \quad (4)$$

where  $G_u$  is the Gibbs function of mixing per unit volume of liquid in the ternary phase (or gel phase):  $G_u = \Delta G_M/(V(1 - \phi_3))$ ,  $V$  being the total volume. Recalling equations (1) and (2), we can write:

$$\begin{aligned} (\partial G_u/\partial u_1)_{u_s} = & (RT/V_1) \{ \ln[u_1/(1 + u_3)] + 1 - s \ln[u_2/(1 + u_3)] \\ & - s + [1/(1 + u_3)](u_2 - u_1)g_{12}(u_1) \\ & + [1/(1 + u_3)]u_1u_2(dg_{12}/du_1) + \\ & + [1/(1 + u_3)]u_3g_{13}(\phi_3) - [u_3/(1 + u_3)]g_{23}(\phi_3) \} \end{aligned} \quad (5)$$

and

$$\begin{aligned} (\partial \Delta G_M/\partial \phi_{10}) = & (RT/V_1) \{ \ln \phi_{10} + 1 - s \ln \phi_{20} - s + \\ & (\phi_{20} - \phi_{10})g_{12}(\phi_{10}) + \phi_{10}\phi_{20}(dg_{12}/d\phi_{10}) \} \end{aligned} \quad (6)$$

$s = V_1/V_2$ , being the ratio between molar volumes of components 1 and 2. Substituting equations (5) and (6) into equation (4) and assuming that  $g_{12}(u_1) \simeq g_{12}(\phi_{10})$  and  $(dg_{12}/du_1) \simeq (dg_{12}/d\phi_{10})$ <sup>9</sup> we arrive at

$$\begin{aligned} \ln(\phi_2/\phi_{20}) = & -m_2\phi_3 \{ \phi_3^{-1} \ln(\phi_{10}/\phi_1) + g_{23}(\phi_3) - g_{13}(\phi_3) - \\ & g_{12}(\phi_{10})[1 + 2(\phi_2 - \phi_{20})/\phi_3] - \{ \phi_1\phi_2/[\phi_3(1 - \phi_3)] \} \\ & (dg_{12}/d\phi_{10}) + \phi_{10}\phi_{20}/\phi_3 \} (dg_{12}/d\phi_{10}) \} \end{aligned} \quad (7)$$

Equation (7) is similar to that obtained by other authors<sup>10</sup> excepting the two last terms of the right hand side, where they use the inadequate parameter  $\chi$ .

Expanding  $\ln(\phi_{10}/\phi_1)$  up to linear terms we can write  $\phi_3^{-1} \ln(\phi_{10}/\phi_1) \simeq 1 + (\phi_2 - \phi_{20})/\phi_3$ . Substituting this expression into equation (7), assuming that  $\phi_{10}\phi_{20} \simeq \phi_1\phi_2$  and neglecting  $\phi_2 - \phi_{20}$  terms, we finally obtain

$$\begin{aligned} \ln(\phi_2/\phi_{20}) = & -m_2\phi_3 \{ 1 + g_{23}(\phi_3) - g_{13}(\phi_3) - \\ & g_{12}(\phi_{10}) - [\phi_{10}\phi_{20}/(1 - \phi_3)](dg_{12}/d\phi_{10}) \} \end{aligned} \quad (8)$$

Alternatively, when there are secondary mechanisms in the g.p.c. separation process, the retention volume of a solute polymer is given<sup>2-4</sup> by:

$$V_R = V_0 + K_D k_p V_i \quad (9)$$

where  $V_0$  is the interstitial or void volume,  $V_i$  is the total volume of solvent within the gel,  $K_D$  is the distribution

coefficient for steric exclusion and  $k_p$  is the distribution coefficient for solute-gel interactions.

For  $k_p$  evaluation, Dawkins and Hemming proposed the expression

$$(V_R - V_0)/k_p = V_i(-A \log M[\eta] + B) \quad (10)$$

in which it was assumed that the hydrodynamic volume is the size parameter determining the steric exclusion separation ( $A$  and  $B$  are constants). A plot of equation (10) as  $\log M[\eta]$  versus the left hand side will give a universal calibration when solutes are separated solely by steric exclusion,  $k_p = 1$ , or when solutes are separated both by steric exclusion and secondary mechanisms (partition and adsorption),  $k_p > 1$ .

The quotient  $\phi_2/\phi_{20}$  in equation (8) is the partition coefficient  $k_p$ , thus we can write

$$\begin{aligned} \ln k_p = & -m_2\phi_3 \{ 1 + g_{23}(\phi_3) - g_{13}(\phi_3) - g_{12}(\phi_{10}) - \\ & (dg_{12}/d\phi_{10})[\phi_{10}\phi_{20}/(1 - \phi_3)] \} \end{aligned} \quad (11)$$

An equation similar to this, differing only in the last term, has already been derived by Horta *et al.*<sup>5</sup>

Since the partition coefficient,  $k_p$ , is evidently related to the sorption of the solute polymer onto the gel (or to the incompatibility between solute polymer and gel) it would be interesting to relate  $k_p$  to the preferential sorption coefficient,  $\lambda$ . The expression for  $\lambda$  at same conditions (finite  $\phi_3$ ) is<sup>11</sup>

$$\lambda(u_1, \phi_3) = \bar{v}_3 A_1 + \phi_3 \bar{v}_3 A_2 \quad (12)$$

where  $\bar{v}_3$  is the specific partial volume of polymer which constitutes the gel matrix,  $A_1 = -M_{13}/M_{11}$  and  $A_2 = -(A_1^2 M_{111} + 2A_1 M_{113} + M_{133})/2M_{11}$ , being

$$\begin{aligned} M_{ij} = & \lim_{u_3 \rightarrow 0} (\partial^2 G_u / \partial u_i \partial u_j)_{u_s} \quad \text{and} \\ M_{ijk} = & \lim_{u_3 \rightarrow 0} (\partial^3 G_u / \partial u_i \partial u_j \partial u_k)_{u_s} \end{aligned}$$

Carrying out the calculations for  $M_{ij}$  and  $M_{ijk}$  we obtain

$$\begin{aligned} (V_1/RT)M_{13} = & s - 1 + g_{13}^\circ - g_{23}^\circ - (\phi_{20} - \phi_{10})g_{12}(\phi_{10}) \\ & - \phi_{10}\phi_{20}(dg_{12}/d\phi_{10}) \end{aligned} \quad (13)$$

$$\begin{aligned} (V_1/RT)M_{11} = & 1/\phi_{10} + s/\phi_{20} - 2g_{12}(\phi_{10}) + \\ & 2(\phi_{20} - \phi_{10})(dg_{12}/d\phi_{10}) + \phi_{10}\phi_{20}(d^2 g_{12}/d\phi_{10}^2) \end{aligned} \quad (14)$$

$$\begin{aligned} (V_1/RT)M_{111} = & -(1 + s) - 6(dg_{12}/d\phi_{10}) \\ & + 3(\phi_{20} - \phi_{10})d^2 g_{12}/d\phi_{10}^2 \end{aligned} \quad (15)$$

$$\begin{aligned} (V_1/RT)M_{113} = & 2g_{12}(\phi_{10}) - 2(\phi_{20} - \phi_{10})(dg_{12}/d\phi_{10}) - \\ & \phi_{10}\phi_{20}(d^2 g_{12}/d\phi_{10}^2) \end{aligned} \quad (16)$$

and

$$\begin{aligned} (V_1/RT)M_{133} = & 1 - s + 2(\phi_{20} - \phi_{10})g_{12}(\phi_{10}) - 2g_{13}^\circ + 2g_{23}^\circ \end{aligned} \quad (17)$$

where  $g_{ij}^\circ$  are the interaction parameters at infinite dilution. Since the selective or preferential sorption of solute polymer (or solvent) onto the gel is irrespectively quantified by equations (11) and (12), both equations must be related.

## RESULTS AND DISCUSSION

As has been already mentioned the partition coefficients,  $k_p$ , have relative characters and their experimental evaluation requires a reference system in which  $k_p = 1$  (steric exclusion). From a thermodynamic point of view, equation (11) and (12) can serve to define a reference state in which the polymers are separated solely by steric exclusion.

At sufficiently low  $\phi_3$  values, equation (11) can be simplified to

$$\ln k_p \simeq -m_2\phi_3(1 + g_{23}^\circ - g_{13}^\circ - g_{12}^\circ) \quad (18)$$

because at these conditions  $g_{ij} = g_{ij}^\circ + \phi_j(dg_{ij}/d\phi_j) \simeq g_{ij}^\circ$  when  $\phi_j \rightarrow 0$ . Furthermore, in equation (18) it has been assumed that  $\phi_2 \simeq \phi_{20} \simeq 0$ , as is usual in chromatographic experiments.

Equation (18) is equal to that deduced by Horta *et al.*<sup>5</sup> (excepting the parameters  $g_{ij}$  which, in our case, are expressed at infinite dilution) and it is also equivalent to the expression obtained by Lecourtier *et al.*<sup>12</sup>.

$$\ln k_p = X_2\{\ln(1 - \phi_3) + \phi_3(\chi_{23} - \chi_{13} - \chi_{12})\} \simeq -X_2\phi_3(1 + \chi_{23} - \chi_{13} - \chi_{12}) \quad (19)$$

The  $k_p$  value predicted by equation (19) is qualitatively similar to that given in equation (18) as long as  $m_2 = V_2/V_1$  is proportional to the degree of polymerization  $X_2$ . Nevertheless, equation (18) is more general than equation (19) because of the dependence of the  $g_{ij}$  parameters on polymer concentration<sup>9</sup>.

The equation for  $\lambda$ , when  $\phi_3 \rightarrow 0$ , is

$$\lambda(u_1) = -\bar{v}_3\{[-1 - g_{23}^\circ + g_{13}^\circ + g_{12}^\circ]/[1 - 2g_{12}^\circ - 2(dg_{12}/d\phi_{10})]\} \quad (20)$$

where  $s$  has been neglected ( $s = V_1/V_2$  and  $V_2 \gg V_1$ ).

On the other hand, the  $g_{23}^\circ$  values can be calculated from<sup>13,14</sup>

$$g_{23}(\phi_3) = [1/(1 - \phi_3)] \int_{\phi_3}^1 \chi_{23}(\phi_3) d\phi_3 \quad (21)$$

considering the variation of  $\chi_{23}$  with  $\phi_3$

$$\chi_{23}(\phi_3) = \chi_{23}^\circ + \chi'_{23}\phi_3 + \chi''_{23}\phi_3^2 + \dots \quad (22)$$

we can write

$$g_{23}(\phi_3) = (\chi_{23}^\circ + \chi'_{23}/2 + \chi''_{23}/3 + \dots) + (\chi'_{23}/2 + \chi''_{23}/3 + \dots)\phi_3 = g_{23}^\circ + g'_{23}\phi_3 + \dots \quad (23)$$

and then

$$g_{23}^\circ = \chi_{23}^\circ + \chi'_{23}/2 + \chi''_{23}/3 + \dots \quad (24)$$

The values of interaction parameters between two polymers,  $\chi_{23}^\circ$ , are usually low and they decrease as molecular weight increases<sup>7,15</sup>. Taking into account that the values of the  $\chi'_{23}$ ,  $\chi''_{23}$ , ... coefficients are also low and that the gel matrix is a crosslinked polystyrene of very high molecular weight,  $g_{23}^\circ$  values can be neglected and equations (18) and (20) can be respectively written as

$$\ln k_p \simeq -m_2\phi_3(1 - g_{13}^\circ - g_{12}^\circ) \quad (25)$$

$$\lambda(u_1) = -\bar{v}_3\{[-1 + g_{13}^\circ + g_{12}^\circ]/[1 - 2g_{12}^\circ - 2(dg_{12}/d\phi_{10})]\} \quad (26)$$

Negative values of  $\lambda$  will indicate preferential sorption of solute polymer (component 2) onto the gel matrix (component 3) and can be interpreted as follows: the macromolecules are 'dissolved' within the 'gel phase' (the macromolecules penetrate inside the grains). The above result is in good agreement with those reached by other authors<sup>12,16</sup>. In these conditions  $k_p > 1$  and  $\ln k_p > 0$ . For usual  $k_p$  values, that is  $1 < k_p < 3^{2-4}$ , and regarding equation (25), two facts arise:

- (1) since  $m_2$  values are high ( $m_2 = V_2/V_1$ ) the corresponding  $\phi_3$  values must be very low as previously mentioned, and
- (2) the  $(1 - g_{13}^\circ - g_{12}^\circ)$  value is negative.

Therefore, in the cyclohexane (CH)/polystyrene (PS)/Styragel (ST) system at 35°C,  $k_p > 1$ <sup>2-4</sup> with a reference system formed by a good solvent for PS/PS/ST. In the former system  $g_{13}^\circ \simeq g_{12}^\circ \simeq 0.705$ <sup>17</sup> and then  $(1 - g_{13}^\circ - g_{12}^\circ) < 0$ . Moreover,  $\lambda$  will be negative because  $-(dg_{12}/d\phi_{10}) \simeq -(dg_{12}/d\phi_1) = (dg_{12}/d\phi_2) \simeq 0.206$ <sup>17</sup> and  $[1 - 2g_{12}^\circ - 2(dg_{12}/d\phi_{10})] > 0$ .

It is interesting to note that the compatibility between the polymers is a direct consequence of the incompatibility between both polymers and the solvent (CH at 35°C is a 'theta' solvent for solute polymer and for the gel matrix polymer).

Contrary to the above is the case of incompatibility between solute polymer and gel matrix. A system satisfying the conditions  $0 < k_p < 1$  and  $\ln k_p < 0$  and  $(1 - g_{13}^\circ - g_{12}^\circ) > 0$  is the benzene (BZ)/PS/ST at 25°C, in spite of the polymers seeming to be compatible. For this system  $g_{13}^\circ \simeq g_{12}^\circ \simeq 0.455$ <sup>9</sup> and  $(1 - g_{13}^\circ - g_{12}^\circ) > 0$ . These values together with  $-(dg_{12}/d\phi_{10}) \simeq -(dg_{12}/d\phi_1) = (dg_{12}/d\phi_2) \simeq 0$ <sup>9</sup> lead to  $[1 - 2g_{12}^\circ - 2(dg_{12}/d\phi_{10})] > 0$  and  $\lambda > 0$  (the solvent is preferentially sorbed into the gel matrix). For this case, although  $\lambda$  values are positive, in absolute terms they are smaller than the above  $\lambda$  negative values.

In the light of the above examples it is clear that the compatibility (or incompatibility) between two polymers must be studied in relation to both polymers, depending on the solvent used. This aspect can be related to the absence of the  $g_{23}^\circ$  parameter in equations (25) and (26).

At this point, an ideal reference system can be defined as a system in which  $(1 - g_{13}^\circ - g_{12}^\circ) = 0$ , with  $\lambda$  also being equal to zero. The former condition is equivalent to that deduced by Horta *et al.*<sup>5</sup>.

Equations (25) and (26) can be applied to chromatographic studies of ternary systems comprised of solvent(1)/polymeric solute(2)/gel matrix(3) if  $k_p$  values and  $g_{12}(\phi_2)$  and  $g_{13}(\phi_3)$  values were known.

In order to carry out the calculations equation (25) can

be written as

$$\ln k_p = -(V_2/V_1)\phi_3[1 - (g_{13}^\circ + g'_{13}\phi_3 + g''_{13}\phi_3^2 + \dots) - g_{12}^\circ] \quad (27)$$

that is to say,  $\phi_3$  is low but not negligible and  $\phi_2 \approx 0$  as is usual in chromatographic experiments. Under these conditions, if the  $\phi_3$  values were known for a system,  $\lambda$  values can be evaluated through equation (12).

Experimentally, the  $\lambda$  variation with molecular weight of solvated polymer is given by<sup>11,18</sup>

$$\lambda M^{1/2} = \lambda_\infty M^{1/2} + A \quad (28)$$

A suitable method of correlating the evaluated  $\lambda$  with the above expression consists of a new definition of the ternary system as solvent(1)/gel matrix(2)/solute polymer(3). Now,  $\phi_3 \approx 0$  (volume fraction of solvated polymer) and  $\phi_2$  is equivalent to  $\phi_3$  in equation (12). With these new conditions the expression for  $\lambda$  is:

$$\begin{aligned} \lambda(u_1, \phi_3) &\approx \lambda(u_1) = -\bar{v}_3(M_{13}/M_{11}) \\ &= -\bar{v}_3[s - 1 + g_{13}^\circ - g_{23}^\circ - (\phi_2 - \phi_1)g_{12}(\phi_1) \\ &\quad - \phi_1\phi_2(dg_{12}/d\phi_1)]/[1/\phi_1 + s/\phi_2 - 2g_{12}(\phi_1) \\ &\quad + 2(\phi_2 - \phi_1)(dg_{12}/d\phi_1) + \phi_1\phi_2(d^2g_{12}/d\phi_1^2)] \end{aligned} \quad (29)$$

Considering that  $g_{12}(\phi_2)$  can be written as  $g_{12}(\phi_2) = g_{12}^\circ + g'_{12}\phi_2 + g''_{12}\phi_2^2 = g_{12}^\circ + g'_{12}(1 - \phi_1) + g''_{12}(1 - \phi_1)^2$  the derivatives will be given by

$$(dg_{12}/d\phi_1) \approx (dg_{12}/d\phi_{10}) = -g'_{12} - 2g''_{12}\phi_2 \quad (30)$$

$$(d^2g_{12}/d\phi_1^2) \approx (d^2g_{12}/d\phi_{10}^2) = 2g''_{12} \quad (31)$$

Substitution of equations (30) and (31) into equation (29) yields

$$\begin{aligned} \lambda(u_1) &= -\bar{v}_3[s - 1 + g_{13}^\circ - (\phi_2 - \phi_1)(g_{12}^\circ + g'_{12}\phi_2 + g''_{12}\phi_2^2) + \\ &\quad \phi_1\phi_2(g'_{12} + 2g''_{12}\phi_2)]/[1/\phi_1 + s/\phi_2 - 2(g_{12}^\circ + g'_{12}\phi_2 + \\ &\quad g''_{12}\phi_2^2) - 2(\phi_2 - \phi_1)(g'_{12} + 2g''_{12}\phi_2) + 2\phi_1\phi_2g''_{12}] \end{aligned} \quad (32)$$

Numerical calculations in this paper have been carried out from literature data as follows:

(1) Considering the ternary system as solvent(1)/solute polymer(2)/gel matrix(3),  $\phi_3$  values have been calculated through equation (27) from  $k_p$  experimental values obtained via g.p.c.

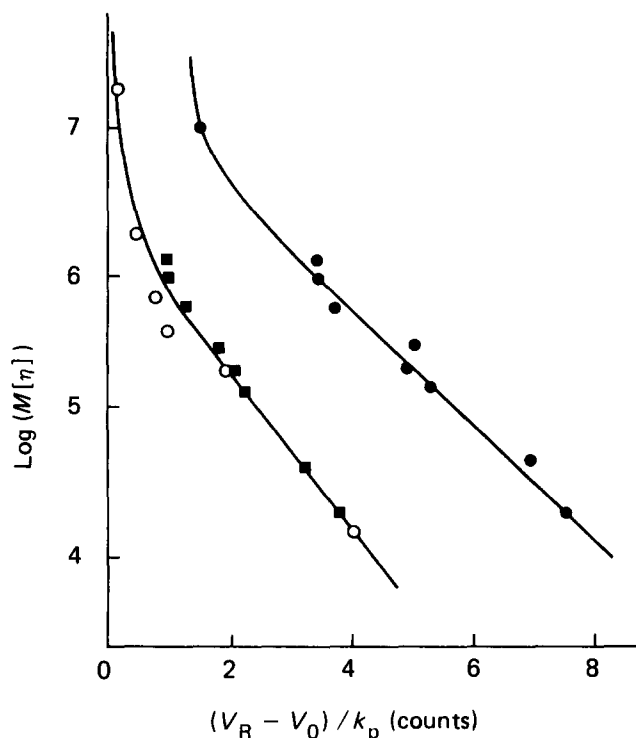
(2) Taking into account the new definition of the ternary system, that is, solvent(1)/gel matrix(2)/solute polymer(3),  $\lambda(u_1)$  values have been calculated through equation (32) from  $\phi_2$  values (that are equivalent to above  $\phi_3$ ).

(3) Equation (28) has been finally applied, with calculated  $\lambda$  values, to each system.

Figure 1 is a plot of  $\log M[\eta]$  versus  $(V_R - V_0)/k_p$  for the systems tetrahydrofuran (THF)/PS/ $\mu$ Styragel ( $\mu$ ST) at 25°C and CH/PS/ $\mu$ ST at 35°C. Table 1 summarizes the results for the latter system. As can be seen, the higher the  $k_p$  values the higher are the molecular weights of the solute polymer. This behaviour agrees qualitatively with that predicted by equation (25). With the  $g_{13}^\circ$  and  $g_{12}^\circ$

values given at the bottom of the table the expression in brackets in equation (25) is negative and so, since  $m_2$  is proportional to the degree of polymerization, predicted  $k_p$  values are higher as the molecular weight of solute polymer increases.  $\phi_3$  values, on the contrary, decrease as molecular weight increases, as Table 1 shows.

On the other hand, negative values of  $\lambda$  indicate a large sorption of the polymer by the gel matrix. The higher the  $\lambda$  values the higher the molecular weight of solvated polymer, this trend being experimentally observed in ternary solvent(1)/solvent(2)/polymer(3) systems<sup>19</sup>. Finally, Table 1 shows the correlation coefficient for the representation  $\lambda M^{1/2}$  versus  $M^{1/2}$  (equation (28)) this



**Figure 1** Universal calibration plot for CH/ $\mu$ ST/PS system at 35°C (●),  $k_p=1$ , THF/ $\mu$ ST/PS at 25°C (○),  $k_p=1$ , and CH/ $\mu$ ST/PS at 35°C (■) with  $k_p$  values for each molecular weight given in Table 1. Experimental data from ref. 6

**Table 1** Experimental  $k_p$ ,  $\phi_3$  (or  $\phi_2$ ) and  $\lambda$  values for CH/ $\mu$ ST/PS system at 35°C. Reference system: THF/ $\mu$ ST/PS at 25°C<sup>6</sup>

$k_p$	$M_w$	$\phi_3 = \phi_2^a$	$\lambda$ (ml g <sup>-1</sup> ) <sup>b</sup>
2.00	3600	0.05385	-0.582
2.19	6000	0.03685	-0.673
2.40	13000	0.01916	-0.774
2.57	15000	0.01792	-0.835
2.80	21000	0.01399	-0.917
2.84	35000	0.00853	-0.940
2.87	49000	0.00616	-0.923
2.90	60000	0.00508	-0.966

Necessary data for calculations:

$g_{13}^\circ$  (equation (27)) =  $g_{12}^\circ$  (equation (32)) = 0.705.

$g'_{13}$  (equation (27)) =  $g'_{12}$  (equation (32)) = 0.206.

$g''_{13}$  (equation (27)) =  $g''_{12}$  (equation (32)) = 0.052.

$g_{12}^\circ$  (equation (27)) =  $g_{13}^\circ$  (equation (32)) = 0.705.

$\bar{V}_1 = 108.74$  ml mol<sup>-1</sup>;  $\bar{V}_2 = M_w = 0.923$  ml mol<sup>-1</sup>;  $\bar{v}_3 = 0.923$  ml g<sup>-1</sup>

<sup>a</sup>  $\phi_3$  (equation (27)) or  $\phi_2$  (equation (32)) values

<sup>b</sup>  $\lambda$  (equation (32)) values

Results obtained through equation (28):  $A = 32.616$ ;  $\lambda_\infty = -1.105$

Correlation coefficient = 0.9992

value indicating that calculated  $\lambda$  values follow the same laws with respect to molecular weight as in the case of ternary solvent(1)/solvent(2)/polymer(3) systems. Moreover, as  $\lambda$  values directly depend on  $\phi_2$  (equation (29))

**Table 2** Experimental  $k_p$ ,  $\phi_3$  (or  $\phi_2$ ) and  $\lambda$  values for CH/ST ( $10^6$ ,  $10^5$ ,  $10^4$  and  $10^3$  Å)/PS system at 35°C. Reference system: CH/ST ( $10^6$ ,  $10^5$ ,  $10^4$  and  $10^3$  Å)/PDMS, PI at 35°C<sup>2-4</sup>

$k_p$	$M_w$	$\phi_3 = \phi_2^a$	$\lambda$ (ml g <sup>-1</sup> ) <sup>b</sup>
1.45	1800	0.05762	-0.336
	3700	0.02845	-0.339
	10300	0.01031	-0.341
	19800	0.00537	-0.341
	51000	0.00209	-0.342
	98200	0.00109	-0.343
	160000	0.00067	-0.342

Necessary data for calculations:  $g_{ij}^0$ ,  $g_{ij}^1$ ,  $g_{ij}^2$ ,  $V_1$ ,  $V_2$  and  $\bar{v}_3$  from Table 1; <sup>a</sup> and <sup>b</sup> as in Table 1

Results obtained through equation (28):  $A = 0.250$ ;  $\lambda_\infty = -0.343$   
Correlation coefficient: 0.999996

**Table 3** Experimental  $k_p$ ,  $\phi_3$  (or  $\phi_2$ ) and  $\lambda$  values for CH/ST( $10^4$  Å)/PS system at 35°C. Reference system: CH/ST( $10^4$  Å)/PDMS, PI at 35°C<sup>2-4</sup>

$k_p$	$M_w$	$\phi_3 = \phi_2^a$	$\lambda$ (ml g <sup>-1</sup> ) <sup>b</sup>
1.60	1800	0.07235	-0.404
	3700	0.03585	-0.419
	10300	0.01303	-0.428
	19800	0.00680	-0.430
	51000	0.00265	-0.433
	98200	0.00137	-0.431
	160000	$8.43 \times 10^{-4}$	-0.432
	411000	$3.28 \times 10^{-4}$	-0.432
	498000	$2.71 \times 10^{-4}$	-0.432
	670000	$2.01 \times 10^{-4}$	-0.432
	867000	$1.56 \times 10^{-4}$	-0.433

Necessary data for calculations:  $g_{ij}^0$ ,  $g_{ij}^1$ ,  $g_{ij}^2$ ,  $V_1$ ,  $V_2$  and  $\bar{v}_3$  from Table 1; <sup>a</sup> and <sup>b</sup> as in Table 1

Results obtained through equation (28):  $A = 0.717$ ;  $\lambda_\infty = -0.433$   
Correlation coefficient: 0.999995

**Table 4** Experimental  $k_p$ ,  $\phi_3$  (or  $\phi_2$ ) and  $\lambda$  values for CH/ST( $10^3$  Å)/PS system at 35°C. Reference system: CH/ST( $10^3$  Å)/PDMS, PI at 35°C<sup>2-4</sup>

$k_p$	$M_w$	$\phi_3 = \phi_2^a$	$\lambda$ (ml g <sup>-1</sup> ) <sup>b</sup>
4.50	1800	0.21550	-0.620
	3700	0.11050	-0.987
	10300	0.04110	-1.233
	19800	0.02159	-1.302
	51000	0.00844	-1.349
	98200	0.00439	-1.362
	160000	0.00270	-1.370
	411000	0.00105	-1.373
	498000	$8.67 \times 10^{-4}$	-1.374
	670000	$6.44 \times 10^{-4}$	-1.374
	867000	$4.98 \times 10^{-4}$	-1.375

Necessary data for calculations:  $g_{ij}^0$ ,  $g_{ij}^1$ ,  $g_{ij}^2$ ,  $V_1$ ,  $V_2$  and  $\bar{v}_3$  from Table 1; <sup>a</sup> and <sup>b</sup> as in Table 1

Results obtained through equation (28):  $A = 18.885$ ;  $\lambda_\infty = -1.401$   
Correlation coefficient: 0.999990

**Table 5**  $\chi_{13}^0$  (equation (34)),  $g_{13}^0$  (equation (35)) and  $g'_{13}$  (equation (36)) values calculated for various systems

System	$K_\theta \times 10^2$	$B \times 10^{28}$	$V_1$ (ml mol <sup>-1</sup> )	$\chi_{13}^0$	$g_{13}^0$	$g_{12}^0$	$g'_{13}$
TD/ST/PS (25°C)	7.296	0.535	157.3	0.497	0.663	0.663	0.166
TD/ST/PS (60°C)	8.350	5.837	167.4	0.465	0.621	0.621	0.155
TD/ST/PS (100°C)	7.470	6.953	176.8	0.457	0.609	0.609	0.152
CH/ST/PS (45°C)	7.584	2.569	110.8	0.490	0.653	0.653	0.163

or  $\phi_3$  (equation (27)), the above result confirms that  $\phi_3$  is not a constant as was previously described<sup>20</sup> but depends on molecular weight of solvated polymer and this variation can affect  $k_p$  values as do  $m_2$  values ( $g_{ij}$  values scarcely vary with molecular weight<sup>13,21,22</sup>).

Analogous results to the above results are shown in Table 2 for the CH/ST ( $10^6$ ,  $10^5$ ,  $10^4$  and  $10^3$  Å)/PS system<sup>2-4</sup> (according to the new nomenclature adopted for the  $\lambda$  evaluation in equation (32)). For this system  $k_p = 1.45^{2-4}$  independently of molecular weight. This result deserves some comment; if  $m_2$  variation is compensated by the  $\phi_3$  variation then equation (25) will yield a  $k_p$  value irrespective of molecular weight and so, in those systems in which  $k_p$  is a constant, large variations in  $\phi_3$  with molecular weight can be expected. As can be seen in Table 2 the variation of  $\phi_3$  with molecular weight is greater than that in Table 1 for the same molecular weight range.

Tables 3 and 4 show  $\phi_3$  and  $\lambda$  values obtained from literature data<sup>2-4</sup> for CH/ST ( $10^4$  Å)/PS and CH/ST ( $10^3$  Å)/PS systems at 35°C. The  $k_p$  values used were 1.6 for the former system and 4.5 for the latter one, and the reference system was the same as in Table 2.  $g_{ij}$  values are given at the bottom of the table. Similar comments to those made in Table 1 can be applied to results collected in these tables, the only difference that stands out is that for equivalent molecular weights, the absolute  $\lambda$  values are lower than those in Table 1. This aspect can be related to the lower specific surface of ST with respect to that of  $\mu$ ST. A good linearity is achieved in both cases when equation (28) is applied, as deduced from the correlation coefficients.

$\phi_3$  and  $\lambda$  values calculated for *trans*-decalin (TD)/ST/PS system at 25°C, 60°C and 100°C are shown in Tables 6, 7 and 8, respectively, with CH/ST/poly(dimethylsiloxane)(PDMS) as the reference system under the same chromatographic conditions.  $g_{ij}$  values are necessary to apply equations (27) and (32), but they are scarce in the literature<sup>9</sup>. To overcome this difficulty the following procedure has been used to evaluate these values. From intrinsic viscosities,  $[\eta]$ , and through the Stockmayer-Fixmann equation

$$[\eta]/M^{1/2} = K_\theta + 0.51\phi_0 B M^{1/2} \quad (33)$$

$K_\theta$  and  $B$  values can be calculated. Table 5 shows  $K_\theta$  and  $B$  values obtained for three temperatures using  $[\eta]$  in ml g<sup>-1</sup> and  $\phi_0 \approx 2.5 \times 10^{23}$ .  $B$  is related to the parameter  $\chi^\circ$  (Flory-Huggins interaction parameter at infinite dilution) through the expression

$$B = (2\bar{v}^2/N_0 V_s)(0.5 - \chi^\circ) \quad (34)$$

where  $\bar{v}$  is the specific partial volume of the polymer,  $N_0$  Avogadro's number and  $V_s$  the molar volume of solvent.  $\chi^\circ$  values calculated through equation (34) are gathered in

**Table 6** Experimental  $k_p$ ,  $\phi_3$  (or  $\phi_2$ ) and  $\lambda$  values for TD/ST/PS system at 25°C. Reference system: TD/ST/PDMS at 25°C<sup>2-4</sup>

$k_p$	$M_w$	$\phi_3 = \phi_2^a$	$\lambda$ (ml g <sup>-1</sup> ) <sup>b</sup>
1.25	1800	$5.797 \times 10^{-2}$	-0.269
	3700	$2.973 \times 10^{-2}$	-0.281
	10300	$1.108 \times 10^{-2}$	-0.290
	19800	$5.823 \times 10^{-3}$	-0.293
	51000	$2.277 \times 10^{-3}$	-0.295
	98200	$1.185 \times 10^{-3}$	-0.295
	160000	$7.28 \times 10^{-4}$	-0.296
	498000	$2.34 \times 10^{-4}$	-0.296
	867000	$1.35 \times 10^{-4}$	-0.297
	1900000	$6.14 \times 10^{-4}$	-0.296

Necessary data for calculations:

$$g_{13}^{\circ} (\text{equation (27)}) = g_{12}^{\circ} (\text{equation (32)}) = 0.663$$

$$g_{13}' (\text{equation (27)}) = g_{13}' (\text{equation (32)}) = 0.166$$

$$g_{13}'' (\text{equation (27)}) = g_{13}'' (\text{equation (32)}) = 0$$

$$g_{12}^{\circ} (\text{equation (27)}) = g_{13}^{\circ} (\text{equation (32)}) = 0.663$$

 $V_1$  from Table 5;  $V_2, \bar{v}_3$  from Table 1;  $a$  and  $b$  as in Table 1Results obtained through equation (28):  $A = 0.713$ ;  $\lambda_{\infty} = -0.297$ 

Correlation coefficient = 0.999996

**Table 7** Experimental  $k_p$ ,  $\phi_3$  (or  $\phi_2$ ) and  $\lambda$  values for TD/ST/PS system at 60°C. Reference system: TD/ST/PDMS at 60°C<sup>2-4</sup>

$k_p$	$M_w$	$\phi_3 = \phi_2^a$	$\lambda$ (ml g <sup>-1</sup> ) <sup>b</sup>
1.20	1800	$6.506 \times 10^{-2}$	-0.209
	3700	$3.397 \times 10^{-2}$	-0.221
	10300	$1.284 \times 10^{-2}$	-0.232
	19800	$6.784 \times 10^{-3}$	-0.236
	51000	$2.662 \times 10^{-3}$	-0.238
	98200	$1.378 \times 10^{-3}$	-0.239
	160000	$8.52 \times 10^{-4}$	-0.239
	498000	$2.74 \times 10^{-4}$	-0.240
	867000	$1.57 \times 10^{-4}$	-0.239
	1900000	$7.2 \times 10^{-5}$	-0.240

Necessary data for calculations:

$$g_{13}^{\circ} (\text{equation (27)}) = g_{12}^{\circ} (\text{equation (32)}) = 0.621$$

$$g_{13}' (\text{equation (27)}) = g_{13}' (\text{equation (32)}) = 0.155$$

$$g_{13}'' (\text{equation (27)}) = g_{13}'' (\text{equation (32)}) = 0$$

$$g_{12}^{\circ} (\text{equation (27)}) = g_{13}^{\circ} (\text{equation (32)}) = 0.621$$

 $V_1$  from Table 5;  $V_2, \bar{v}_3$  from Table 1;  $a$  and  $b$  as in Table 1Results obtained through equation (28):  $A = 0.882$ ;  $\lambda_{\infty} = -0.241$ 

Correlation coefficient = 0.999994

Table 5. Once the  $\chi^{\circ}$  parameters are known,  $g_{13}^{\circ}$  can be evaluated from<sup>23</sup>

$$\chi_{13}^{\circ} = [1 - (2/Z)]g_{13}^{\circ} \quad (35)$$

where  $Z$  is the coordination number of the network. A value of  $Z = 8$ <sup>24</sup> has been used.  $g_{13}^{\circ} = g_{12}^{\circ}$  because both the solute and the gel are the same polymer and taking into account the equation of Koningsveld *et al.*<sup>13,17,21,22</sup> we can write

$$g_{12}' = g_{13}' = (dg_{13}/d\phi_3)_{\phi_3 \rightarrow 0} = g_{13}^{\circ} - \chi_{13}^{\circ} \quad (36)$$

Table 5 shows  $g_{13}^{\circ}$ ,  $g_{12}^{\circ}$  and  $g_{13}'$  values for each system. There is no information concerning  $g_{13}''$  values in the bibliography, however, they always are multiplied by  $\phi_j^2$  and as  $\phi_j$  is very low, their values can be neglected in any calculations.

$\lambda$  values gathered in Tables 6, 7 and 8 completely fulfil equation (28) as the correlation coefficients indicate. Also it can be noted that, for the same molecular weight, the  $\lambda$  values decrease when the temperature is raised, that is to

say the secondary mechanisms are lower as temperature is higher.

Finally, in Table 9 results for the CH/ST(10<sup>3</sup> Å)/PS system at 45°C<sup>2-4</sup> are shown,  $k_p = 2$  with PDMS as the reference system.  $g_{ij}$  values necessary for application of equations (27) and (32) are given in Table 5. As can be seen  $\lambda$  values follow equation (28) well as the correlation coefficients indicate. If results of Table 9 are compared with those of Table 4 (the same system but at 35°C)  $k_p$  values fall from 4.5 to 2.0 and  $\lambda$  diminishes in absolute value. This result again confirms that the lower the  $\lambda$  values the lower the importance of the secondary mechanisms.

Throughout the tables, it can be observed that  $k_p$  is not always a constant. As other authors confirm<sup>24</sup>,  $k_p$  values may vary with molecular weight. As has been already mentioned  $k_p$  and  $\lambda$  must be related. In this way, we have found a variation for  $k_p$  similar to that of equation (32) for  $\lambda$ . That is to say,  $k_p M^{1/2}$  vs.  $M^{1/2}$  is a linear correlation. The study of this variation will be subject of a subsequent paper.

**Table 8** Experimental  $k_p$ ,  $\phi_3$  (or  $\phi_2$ ) and  $\lambda$  values for TD/ST/PS system at 100°C. Reference system: TD/ST/PDMS at 100°C<sup>2-4</sup>

$k_p$	$M_w$	$\phi_3 = \phi_2^a$	$\lambda$ (ml g <sup>-1</sup> ) <sup>b</sup>
1.10	1800	$4.168 \times 10^{-2}$	-0.136
	3700	$2.136 \times 10^{-2}$	-0.134
	10300	$7.955 \times 10^{-3}$	-0.134
	19800	$4.181 \times 10^{-3}$	-0.135
	51000	$1.635 \times 10^{-3}$	-0.135
	98200	$8.51 \times 10^{-4}$	-0.135
	160000	$5.23 \times 10^{-4}$	-0.135
	498000	$1.68 \times 10^{-4}$	-0.135
	867000	$9.7 \times 10^{-5}$	-0.135
	1900000	$4.4 \times 10^{-5}$	-0.135

Necessary data for calculations:

$$g_{13}^{\circ} (\text{equation (27)}) = g_{12}^{\circ} (\text{equation (32)}) = 0.609$$

$$g_{13}' (\text{equation (27)}) = g_{13}' (\text{equation (32)}) = 0.152$$

$$g_{13}'' (\text{equation (27)}) = g_{13}'' (\text{equation (32)}) = 0$$

$$g_{12}^{\circ} (\text{equation (27)}) = g_{13}^{\circ} (\text{equation (32)}) = 0.609$$

 $V_1$  from Table 5;  $V_2, \bar{v}_3$  from Table 1;  $a$  and  $b$  as in Table 1Results obtained through equation (28):  $A = -0.020$ ;  $\lambda_{\infty} = -0.135$ 

Correlation coefficient = 0.999994

**Table 9** Experimental  $k_p$ ,  $\phi_3$  (or  $\phi_2$ ) and  $\lambda$  values for CH/ST/PS system at 45°C. Reference system: CH/ST/PDMS at 45°C<sup>2-4</sup>

$k_p$	$M_w$	$\phi_3 = \phi_2^a$	$\lambda$ (ml g <sup>-1</sup> ) <sup>b</sup>
2.00	1800	$1.202 \times 10^{-1}$	-0.390
	3700	$6.455 \times 10^{-2}$	-0.485
	10300	$2.504 \times 10^{-2}$	-0.565
	19800	$1.334 \times 10^{-2}$	-0.591
	51000	$5.268 \times 10^{-3}$	-0.610
	98200	$2.751 \times 10^{-3}$	-0.616
	160000	$1.692 \times 10^{-3}$	-0.619
	498000	$5.45 \times 10^{-4}$	-0.621
	867000	$3.13 \times 10^{-4}$	-0.622
	1900000	$1.43 \times 10^{-4}$	-0.622

Necessary data for calculations:

$$g_{13}^{\circ} (\text{equation (27)}) = g_{12}^{\circ} (\text{equation (32)}) = 0.653$$

$$g_{13}' (\text{equation (27)}) = g_{13}' (\text{equation (32)}) = 0.163$$

$$g_{13}'' (\text{equation (27)}) = g_{13}'' (\text{equation (32)}) = 0$$

$$g_{12}^{\circ} (\text{equation (27)}) = g_{13}^{\circ} (\text{equation (32)}) = 0.653$$

 $V_1$  from Table 5;  $V_2, \bar{v}_3$  from Table 1;  $a$  and  $b$  as in Table 1Results obtained through equation (28):  $A = 6.168$ ;  $\lambda_{\infty} = -0.628$ 

Correlation coefficient = 0.99997

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